Mathematical Understanding of Nature: Essays on Physical Phenomena and Their Understanding by Mathematicians by V. I. Arnold, translated by Alexei Sossinsky and Olga Sipacheva

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Mathematical Understanding of Nature: Essays on Physical Phenomena and Their Understanding by Mathematicians

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REVIEWED BY MARK LEVI

Vladimir Igorevich Arnold (1937–2010) was one of the best known and most influential mathematicians of the twentieth century, having made seminal contributions in many areas of mathematics, including dynamical systems, singularity theory, and hydrodynamics. His expository talent contributed to his fame. His writing was concise, with not an ounce of verbal fat, with ideas up front, formulated with great appeal, frequently with simple geometrical illustrations.

Arnold's laconic writing style was loved by many; others found his writing difficult to understand for a perceived lack of sufficient detail. The famous *Feynman Lectures on Physics* elicit similarly polarized reactions: some (including this writer) love them and some don't. Arnold paid great attention to the aesthetic appeal of his writing, as I observed when we met, back in the early 1990s. He complained that the labeling of sections in the translation of his book [1], of which I was the editor, was changed from the original labeling to include chapter numbers, which made the numbering too busy, in his opinion (with which I agreed). I also wondered sometimes whether he considered being boring (which he never was) to be worse than being wrong (which on rare occasions he was, as are most of us.).

The book under review, a translation of the original Russian edition, which appeared in 2011, is a collection of 39 short chapters (fewer than four pages each on average), each discussing a problem, usually from basic physics and containing a mathematical kernel. There are many books on popular physics and on popular mathematics, with emphases leaning toward one subject or the other (the famous classic [3] by Littlewood comes to mind.) At the purely physical end of the spectrum are books like Walker's *Flying Circus of Physics* [5] and Minnaert's fascinating book on light [4]. Arnold's book is driven by applications, but with mathematical ideas at the core, and is very entertaining. With a sprinkling of personal anecdotes and historical asides, the essays did not come even close to taxing my attention span.

Who will this book appeal to? The scientific level of the essays is quite nonuniform. Some require just basic geometry; others will probably be accessible only to relatively advanced readers. But the book abounds in fascinating nonmathematical digressions and historical references that would pull in even a nonmathematician. Without exception, the problems in this book will appeal to those who are interested in physical aspects of mathematics. A few provocative remarks (on the bad state of education, on the alleged narrow-mindedness of some people) are sprinkled about, adding some spice (which may taste bitter to some readers). I think that this book would be a great addition to a faculty lounge; it would certainly provoke discussions.

In Chapter 25, on Bernoulli fountains, Arnold may be pulling the reader's collective leg. Water jets three meters high gush out of the drain holes of a bridge during a storm, he says, as predicted by Bernoulli's law: the wind is much faster, and thus the air pressure much lower, above the bridge than below it. This suction, he says, caused the jets. But isn't three meters a bit high?

Indeed, the pressure required to shoot a water jet to such a height is 30% of the atmospheric pressure! How fast must the wind blow to yield such a pressure difference? A quick estimate yields the preposterous 250 meters per second, still subsonic but not by much—about the speed of a jetliner. Such a speed would blow away the bridge along with the proposed explanation. Parenthetically, the 250 m/sec estimate comes from Bernoulli's principle that $\rho v^2/2 + p$ is constant. (Here ρ is the air density, which is assumed to be constant, v is the wind speed, and p is the pressure. And this value is constant not just along a particle's path but throughout, as the explanation assumes implicitly; this amounts to the assumption of irrotational flow.) With the air speed $v_0 \approx 0$ under the bridge and with the larger speed v over the bridge, we would have

$$\rho v_0^2/2 + p_{\text{atm}} = \rho v^2/2 + 0.7 p_{\text{atm}}.$$

Solving for v and substituting $\rho \approx 1\,\text{kg/m}^3$ (air is surprisingly heavy) and $p_{\text{atm}} = 10^5\,\text{N/m}^2$ gives $v \approx 250\,\text{m/sec}$. Of course, the 30% pressure difference invalidates the assumption of constant density, but the point of this estimate remains valid.

Another problem certain to attract considerable interest concerns a bicycle with one pedal placed in its lowest position and pulled backward. With the dimensions of all gears and the wheel given, one is asked which way the pedal will move. Arnold presents a calculation according to which the bike will move forward, and not only the bike, but the pedal itself (relative to the ground). The latter claim is incorrect, and it does not require calculation to see why. Indeed, with the bike standing on a table we may hang a brick tied to the pedal by a string thrown over a pulley, as shown in Figure 1.The string will pull the pedal backward, causing it—if the claim were correct—to move forward, raising the brick. But that is against the law of conservation of energy: the potential energy of the bike–brick–Earth system would spontaneously increase, with no outside input.

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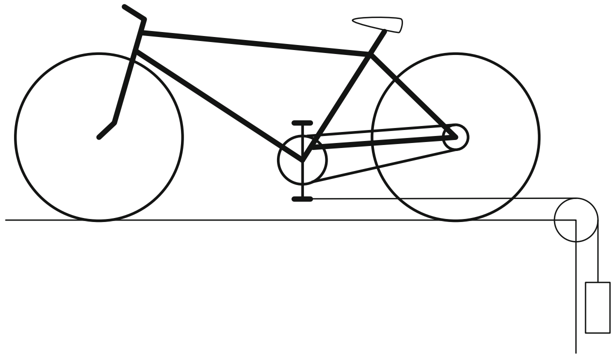

Figure 1. The forward motion of the pedal relative to the ground would raise the brick, violating the law of conservation of energy.

The editors of the second Russian edition note that although the error was pointed out by several readers, they chose to leave the problem and the solution in their original form, since Arnold had died suddenly while the new edition was in preparation and could therefore not contribute to a correction—an excellent decision, I think. Mistakes make problems much more entertaining, as Arnold himself had said. And the more prominent the culprit, the more entertaining the mistake (and the more liberating for the rest of us). In a concluding remark, the editor suggests changing the problem by placing the pedal puller on the bike seat rather than on the ground. Unfortunately, this reformulation was almost certainly not what Arnold intended: the problem is interesting only because the answer is so counterintuitive. Actually, Arnold's answer is correct in reference to the bike itself rather than the pedal, and so the problem is still quite interesting. And in fact, Arnold's error made it perhaps even more interesting. And sometimes, an error is best left uncorrected, which reminds me of a story about a boy who had gotten his head stuck in a tall Chinese vase. As they were riding in a crowded streetcar on the way to the hospital emergency room, the boy's mortified mother tried to make her son look more presentable by placing a hat on top of the upturned vase.

Returning to the book under review, I note that several problems could have been treated better. The stability of an inverted pendulum with vibrating support is left unexplained (although a calculation that proves the effect but does not explain it is referred to).

One of the mini-chapters discusses reflections in mirrors. As we look at our reflection in a shiny sphere (or rather a circle, confining ourselves to the plane), what is the location of the image inside the circle? Arnold shows that the image is the inversion of the object with respect to the circle tangent to the mirror and centered at the midpoint of the radius directed at the observer. The proof of this fact is presented, although it could perhaps have been made simpler by replacing some calculations with more conceptual arguments. There are other gems in this description, for instance the observation on a caustic inside the reflecting circle (it could have been mentioned that this caustic inside a mirror of radius r is a hypocycloid obtained by rolling a circle of radius $r / 4$ on the circle of radius $r / 2$ concentric with the mirror).

Arnold's terse style is a plus for some readers and a minus for others; an individual reader may even have a mixed reaction. The advantage of brevity is that the details are omitted, so that the ideas stand out; the disadvantage is that the details are omitted. Consider Arnold's explanation of mirages. A figure on page 26 shows the inverted mirage image of a distant palm tree; Arnold explains that the inversion is due to the fact that a ray from the top of the tree enters the eye below a ray from the bottom of the tree. Those familiar with basic optics will realize that Arnold is referring to the direction of the ray rather than the point of entry—we perceive the direction of rays entering our eyes. And in any case, it is not one ray but a pencil of rays that illuminates a "pixel" on the retina of a focused eye. I am afraid some readers will not catch these points, but they will be stimulated to figure them out on their own!

Some chapters, in particular on integral geometry and on ergodic theory, explain absolute gems in a simple nontechnical way that I have not seen done by any other author.

The last chapter, "On Rotation of Rigid Bodies and Hydrodynamics," gives a fascinating historical perspective of the subject of the chapter's title. To quote the author:

> Scrutinizing Euler's treatise on the Moon's rotation in 1965 on the occasion of its bicentenary, I noticed that Euler's arguments prove much more than Euler stated. Namely, his whole theory carries over, almost without changes, to the study of geodesic lines on Lie group manifolds endowed with a left- (or right-) invariant Riemannian metric.

This observation led to Arnold's breathtakingly beautiful work on hydrodynamics of ideal fluids, establishing, in particular, the intimate connection between the stability of rotations of a rigid body and Rayleigh's stability criterion for ideal fluids [2].

In summary, this is a very entertaining and informative book. Even the faults (some of which I have mentioned, and some which I have not) add to the book's appeal.

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