A Sprinkler Paradox

MATHEMATICAL CURIOSITIES
By Mark Levi

With warmer weather hopefully coming to the Northeastern U.S.[1] I would like to introduce a sprinkler-related question that occurred to me some time ago [1]. The sprinkler in Figure 1 is an S-shaped tube that pivots on a frictionless hinge, with a steady supply of water to the hinge. Water then shoots out of both ends of the tube, causing it to spin. The sprinkler was activated in the sufficiently distant past and is now in steady state.

1. What is the direction of the velocity of the exiting water (viewed by a ground observer from the top)?

2. What is the ground speed of the exiting water? Is it equal to the speed in the tube?

3. In what direction does water exit a sprinkler with semicircular arms, as in Figure 2?

[1] This article was written in mid-April.

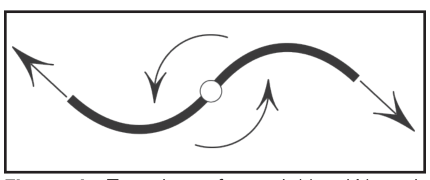

Figure 1. *Top view of a sprinkler. Water is supplied at the center hub and shoots out of the ends of the S-tube, causing it to spin.*

4. What shape(s) of the S-tube will maximize the angular velocity? Is it good to maximize the velocity?

5. What is the shape of the water jet (as viewed from the top by a ground observer) after it exits the sprinkler? What is the shape of the same jet when viewed by an observer who spins with the sprinkler?

Here are answers to some of these questions.

1. Despite the sprinkler's spin, the ground velocity of exiting water is perfectly radial (see Figure 3). Indeed, the angular momentum of water that is supplied at the hinge is zero, and no external torque is applied to the water-tube system because the hinge is frictionless. Exiting water hence has zero angular momentum as well. In short, since the motion is in steady state, angular momentum in equals angular momentum out. And since the former is zero, so is the latter.

2. Exit speed u (relative to the ground) is slower than the speed v in the tube:

$$u = v\cos\theta, \qquad (1)$$

where θ is the angle between the tangent at the tube's end and the radial ray (see Figure 3).

3. For the sprinkler with semicircular arms in Figure 2, the water exits with zero ground speed and just plops to the ground. This is evident from (1)—with $\theta = \pi/2$—or directly from Figure 3. So, the sprinkler in Figure 2 would only irrigate a circle.

I leave questions 4 and 5 for the reader's possible amusement.

And as a concluding puzzle, the water that exits the "bad" sprinkler in Figure 2 has zero kinetic energy. But the water that enters the sprinkler certainly has positive kinetic energy. What happened to the kinetic energy? The same question applies to the more general sprinkler in Figure 1, since the exit speed $u < v$.

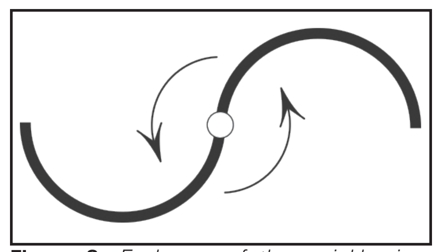

Figure 2. *Each arm of the sprinkler is a semicircle. Water has zero ground speed upon exiting the tube.*

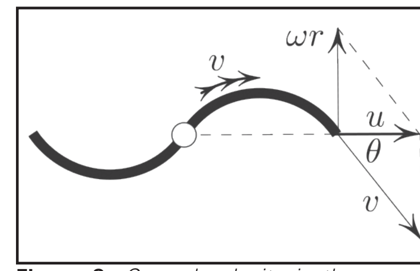

Figure 3. *Ground velocity is the sum of two velocities and points radially away from the pivot.*

References

[1] Levi, M. (2012). *Why cats land on their feet: And 76 other physical paradoxes and puzzles*. Princeton, NJ: Princeton University Press.

The figures in this article were provided by the author.

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