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GYROSCOPIC EFFECTS IN A ROTATING SLEEVE HYDROCYCLONE

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Abstract—In this short note, we give a simple explanation of a paradox observed in the rotating sleeve hydrocyclones based on the hypothesized existence of a Taylor column. A simple test to verify the existence of such a column is described. The effect of such a column upon the efficiency of the device is described, together with a suggested improvement of the design.

THE BACKGROUND

Rotating sleeve hydrocyclones are used in the oil industry to separate oil droplets from the water/oil mixture in situations where space is at a premium [1]. The device consists of the outer cylindrical tube (the sleeve) which is in sliding contact with the endcaps; the contact allows the sleeve to rotate (see Figure 1). The water/oil mixture is injected through one endcap through the opening far from the axis of the cylinder. The shape of the opening is such that the mixture comes into the cylinder approximately tangentially to a circle in the plane perpendicular to the axis of the cylinder, as shown in Figure 1, so that the entering mixture swirls around the cylinder. The sleeve may rotate at a speed which is on the same order of magnitude as the swirling speed of the injected mixture. The bouyancy caused by the centrifugal effect draws the oil to the central core, and the oil comes out through the central openings of the two endcaps (Figure 1).

In the mathematics/industry workshop at Rensselaer, in May 1992, Dale Larson [2] from Chevron Research had mentioned several unexplained phenomena in the behavior of the device. One such phenomenon is observed when the rotation axis of the hydrocyclone is vertical and the water/oil mixture is injected at the top, with the water exiting at the bottom. The average downward speed of the water is on the order of 20 ft/sec. The core of oil rises relative to the surrounding fluid at the much slower rate. (This can be easily estimated from the knowledge of the buoyancy of oil, the thickness of the oil core (about .25 cm), and from the viscosities of the two fluids.) One would naturally expect the oil to be swept downwards and out of the lower center hole. Remarkably, just the opposite happens: the oil comes out of the top, against the much faster flow of water! The aim of this note is to give a possible explanation of this effect and to propose a simple experiment to verify the validity of this explanation. For the reader's convenience, we recall the following theorem.

THE TAYLOR-PROUDMAN THEOREM

Consider a fluid motion in a frame of reference rotating with a constant angular velocity Ω. The Euler's equations of motion in this frame are

$$u_t + u \cdot \nabla u = -\nabla \left(\frac{p}{\rho} - \frac{1}{2} |\Omega \times r|^2 \right) + 2u \times \Omega,$$

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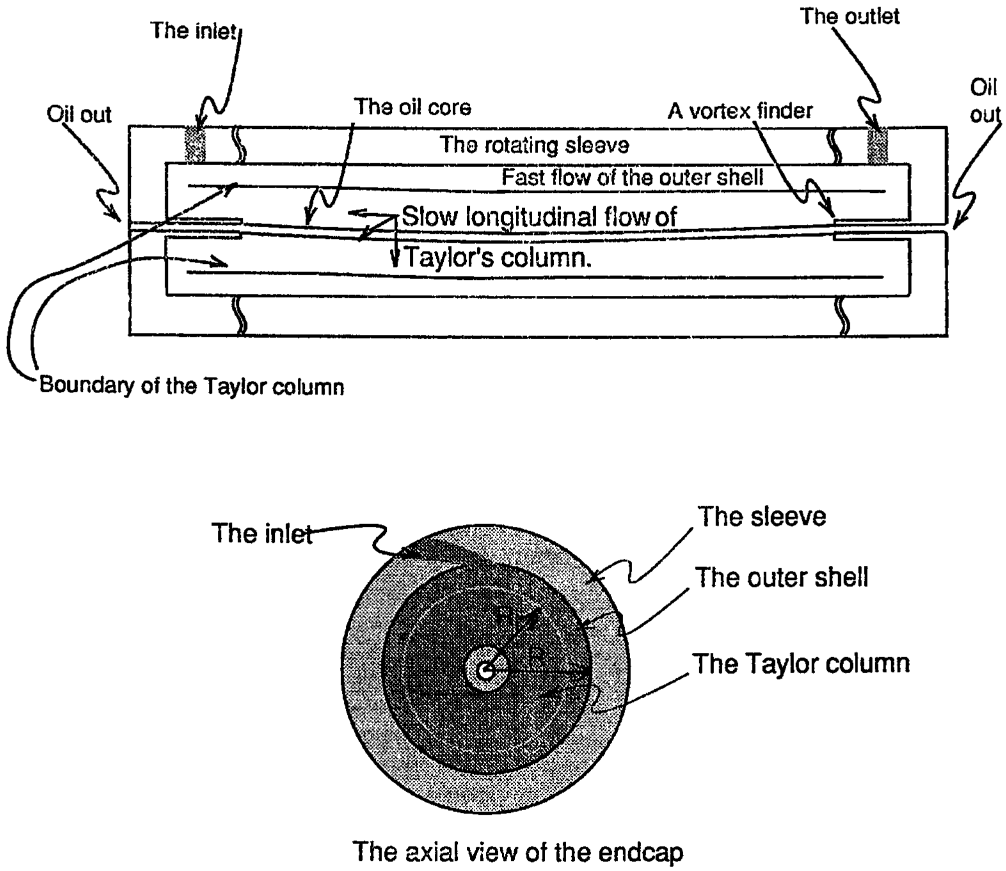

Figure 1. The rotating sleeve hydrocyclone—a schematic drawing.

where the two terms containing Ω are the centrifugal and the Coriolis forces, respectively. Assuming u to be small with respect to Ω, we neglect the term $u \cdot \nabla u$, and taking the curl of the last equation, we obtain an approximation

$$\omega_t - 2\,\mathrm{curl}(u \times \Omega) = 0.$$

If Ω is large in comparison with ω and ω_t, we may keep only the leading term in this equation:

$$\mathrm{curl}(u \times \Omega) = 0.$$

Observing the identity

$$\mathrm{curl}(u \times \Omega) = \Omega \cdot \nabla u + \Omega\,\mathrm{div}\,u,$$

(valid for $\Omega = \mathrm{const}$) and using $\mathrm{div}\,u = 0$, we reduce the previous equation to

$$\Omega \cdot \nabla u = 0.$$

This in effect is the Taylor-Proudman theorem. To read off its physical meaning, we note that according to the last equation, Ω is the eigenvector of the matrix $\nabla u = \frac{\partial u}{\partial x}$, or equivalently, Ω is the line of stationary equilibria for the linearized vectorfield $\dot{\xi} = \frac{\partial u}{\partial x}\,\xi$. This is equivalent to saying that the lines in the direction Ω behave as "stiff wires": they resist bending as well as contraction or stretching. This can be explained directly by analyzing the effect of the Coriolis force (precisely the term we did not omit) as done, e.g., in [3]. Further details and discussion can be found in [3–5].

IMPLICATIONS FOR THE ROTATING SLEEVE HYDROCYCLONE

Let us choose the rotating frame rigidly attached to the spinning sleeve. The endcaps are rotating in this frame; referring to the figure, we consider the annular region A, which is the longitudinal projection of the first helical loop of the incoming jet (Figure 1). We will refer to the cylindrical shell with the annular base A as the *outer shell* and to the cylinder inside that shell as the *inner cylinder*.

If the speed of injected water/oil mixture coincides approximately with the linear speed of the rotating sleeve, we can make a rough assumption that the speed of the fluid in our rotating frame is small in comparison with the angular velocity Ω of the sleeve, and thus the Taylor-Proudman phenomenon should manifest itself, causing the vortex lines to "stiffen." The vortex lines are, at least away from the boundaries of the cylinder and of the outer shell and from the endcaps, approximately straight lines parallel to the axis. Those lines which lie in the inner cylinder impinge on the wall (with the exception of the narrow core cylinder based at the oil outlet openings), and thus the longitudinal motion is precluded for those vortex lines according to the Taylor-Proudman theorem. One thus might expect most of the longitudinal motion to be concentrated in the outer shell, Figure 2. In other words, *the fluid in the inner cylinder forms a Taylor column.* This explains the apparent paradox of the oil rising to the top vortex finder without being swept down with the fast flow. One can perform the following experiment to verify the validity of the predicted velocity profile.

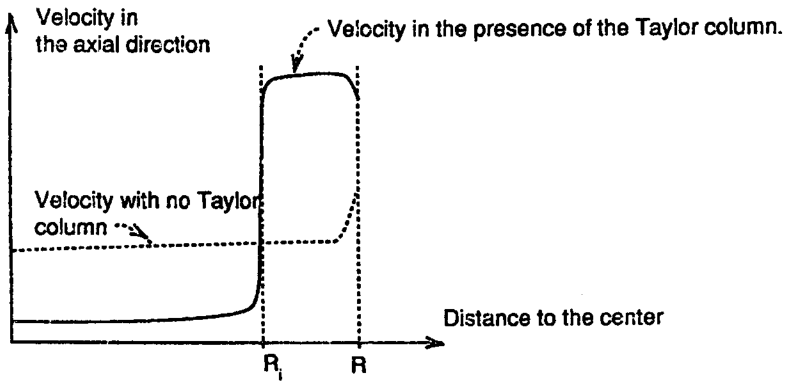

Figure 2. The velocity profile based on the the existence of the Taylor column.

AN EXPERIMENT

By injecting a short burst of dye, one can record its passage time T. A finding of

$$T \approx \frac{\pi(R^2 - R_i^2)L}{F} \equiv \frac{1}{F}\,(\text{volume of the outer shell}), \tag{1}$$

where F is the flux, would confirm the above prediction of the velocity profile. In deriving equation (1), we used the relationship $T = L/v$. If, on the other hand, the longitudinal profile were uniform, then the passage time would be longer than that given by (1), namely

$$T_{\text{uniform}} = \frac{\pi R^2 L}{F} = \frac{1}{F}\,(\text{volume of the sleeve}). \tag{2}$$

THE TAYLOR COLUMN AND THE SEPARATION EFFICIENCY

We will refer to the proportion of the separated oil to the entering oil as the *efficiency*, in accordance with the common usage and sense. How does the Taylor column affect the efficiency? To answer this question, we will derive two different estimates of the efficiency based on two conflicting assumptions (of which we expect the second one to be closer to the truth):

ASSUMPTION 1. *The longitudinal velocity profile is approximately uniform at least away from the endcaps.*

ASSUMPTION 2. *The longitudinal velocity profile is as dictated by the existence of the Taylor column (see Figure 2).*

To calculate the efficiency under Assumption 1, we first focus our attention on the oil droplets of a certain fixed radius a. Consider a thin disk of fluid near the left endcap (Figure 2) as it travels without distortion along the axis while rotating around the axis at the same time. The centrifugal effect activates the buoyancy of oil droplets which gravitate towards the center of the disk. Let R_0 be the maximal radius within which the particles will enter the oil core by the time the other end is reached. The efficiency is given by

$$E_1 = \frac{\pi(R_0^2 - R_{\text{core}}^2)}{\pi(R^2 - R_{\text{core}}^2)} \approx \left(\frac{R_0}{R}\right)^2, \tag{3}$$

and it remains to compute R_0. Let $r(t)$ be the distance of an oil droplet from the axis of the cylinder. We have

$$k\dot{r} = -m\Omega^2 r,$$

where

$$m = ((\text{density of the water}) - (\text{density of the oil})) \times \frac{4}{3}\pi a^3,$$

$\Omega =$ the angular velocity of the rotating disk and k is the coefficient of drag given by the Stokes formula [3]: $k = 6\pi\,\mu\,a$. We now determine R_0 from the conditions $r(0) = R_0$ and $r(T) = R_{\text{core}}$, where $T = \frac{L}{v}$ is the travel time from the left to the right and v is the longitudinal velocity. Substituting all these data into the solution $r(t) = r(0)\,e^{-ct}$, we obtain $R_0 = \exp\left(\frac{m\Omega^2}{k}\frac{L}{v}\right) R_{\text{core}}$, and equation (3) reduces to

$$E_1 = \left(\frac{R_{\text{core}}}{R}\right)^2 e^{(2m\Omega^2 L/kF)\,S}, \tag{4}$$

where F is the flux, $S = \pi R^2$ is the cross-sectional area of the interior of the sleeve; we observe that $F = Sv$, using Assumption 1 on the uniform longitudinal velocity.

In the same way, from Assumption 2 on the existence of a Taylor column with the cross-sectional area equal to $s < S$, we obtain the efficiency estimate

$$E_2 = \left(\frac{R_{\text{Taylor column}}}{R}\right)^2 e^{(2m\Omega^2 L/kF)\,s}. \tag{5}$$

The key point here is that once the oil drop enters the Taylor column, it is likely to spend a long time there, since the longitudinal velocity of the column is small, and thus is most likely to be captured by the oil core. We obtain the desired ratio

$$\frac{E_2}{E_1} = \left(\frac{R_{\text{Taylor column}}}{R_{\text{oil core}}}\right)^2 e^{(2m\Omega^2 L/kF)\,(s-S)}, \tag{6}$$

but it remains to observe that the angular velocity Ω due to the "swirl" of injected mixture and the flux F are roughly proportional:

$$\Omega \approx \frac{v_{\text{injected}}}{R} = \frac{F}{\sigma R},$$

where σ is the cross-sectional area of the inlet. Substitution into equation (6) gives

$$\frac{E_2}{E_1} = \left(\frac{R_{\text{Taylor column}}}{R_{\text{oil core}}}\right)^2 e^{(2mL(s-S)/k\sigma^2 R^2)\,F}. \tag{7}$$

A REMARK ON A POSSIBLE DESIGN IMPROVEMENT

One can draw some predictions and recommendations from the above estimate. As an example, the estimate shows that for large values of F, the Taylor column has a detrimental effect on the efficiency. This suggests that changing the design in such a way as to make uniform the longitudinal velocity profile might improve the efficiency. For instance, *changing the geometry and location of the inlet so as to allow the Taylor column to move in the axial direction* might, in fact, help uniformize the longitudinal velocity and thus improve the efficiency. To that end, one can shape the outlet opening so as to extend it in the radial direction (so that the intake is not just at the rim of the endcap).

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