

Further Upon Reflection: Caustics in a Ball

What can one say about the location of the sun's reflection in a shiny ball? Before discussing the answer, let us first consider a circular mirror with a reflecting inner (rather than outer) surface (see Figure 1) illuminated by a pencil of incoming parallel rays — like the top view of a cup in the sun. The bright line on the bottom of the cup is the envelope of the family of reflected rays; such envelopes are called *caustics*.

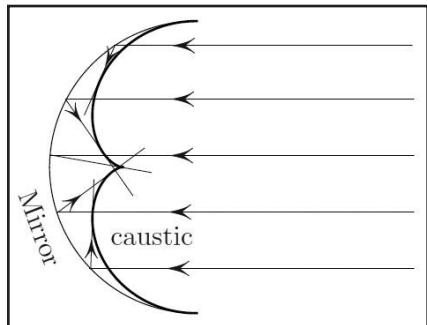


Figure 1. A circular mirror “focuses” the incoming beam on a caustic.

Interestingly, the exact same caustic is the locus of the sun's reflections from the *outside* surface of the same circular mirror (see Figure 2). The caustic in Figure 2 is the envelope of (the lines defined by) reflected rays. Since the same words describe the caustic in Figure 1, the two caustics are indeed identical; Figure 3 depicts the outer and inner reflections side-by-side, illustrating the identity of the two caustics in one picture.

Now, why do the sun's reflections lie on the caustic? Figure 2 gives the answer: each point (such as *a* or *b*) on the caustic/enve-

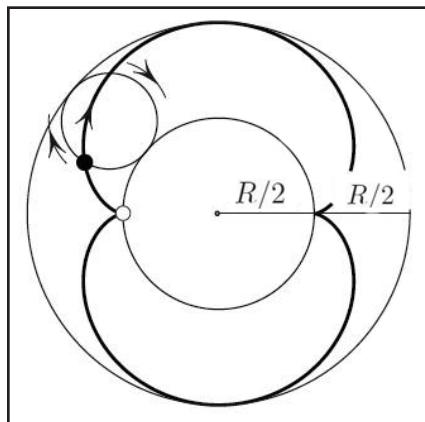


Figure 4. The nephroid is a hypercycloid traced by a point of a wheel rolling on a circle, with the 1:2 ratio of the radii.

lope acts, infinitesimally speaking, as the point source of light. To the observer *A*, it will thus appear as if a point source of light exists at *a*. To summarize, every reflection in any shiny mirror lies on a caustic, i.e., on the envelope of reflected rays. This holds for a mirror of any shape, not only a circle.

MATHEMATICAL CURIOSITIES

By Mark Levi

For the circular mirrors in Figures 1 and 2, the caustic happens to be a nephroid, i.e., a hypercycloid traced by a point on the rolling wheel, as shown in Figure 4. I provided a short proof of this fact in my April¹ column.

The figures in this article were provided by the author.

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¹ <https://sinews.siam.org/Details-Page/focusing-on-nephroids>

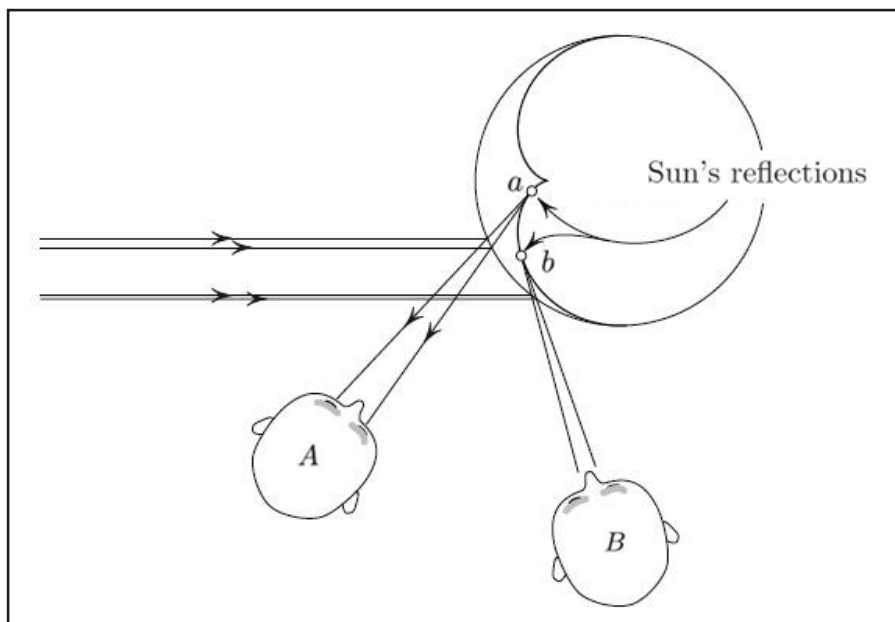


Figure 2. As the observer walks around the circular mirror, the sun's reflections slide along the caustic.

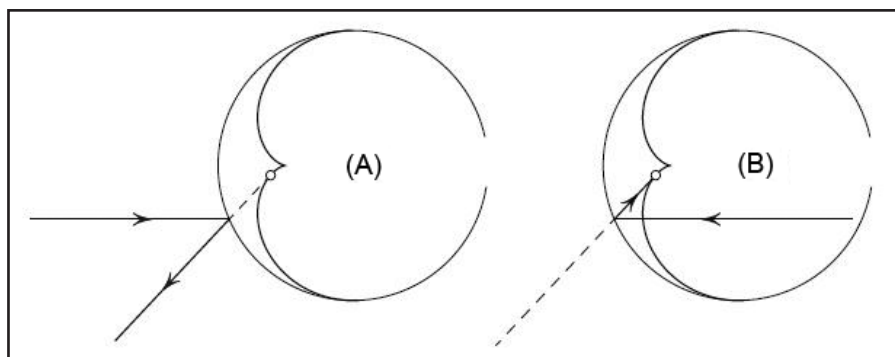


Figure 3. The locus of the sun's images in a circular mirror (A) coincides with the caustic on the bottom of a coffee cup (B).