Downwind, Faster Than the Wind

By Mark Levi

The feasibility of the title's suggestion depends on one's definition of sailing. A regular sailboat cannot exceed wind speed when going dead downwind, i.e., exactly in the wind's direction. But if propellers and gears are used instead of sails, then the seemingly impossible becomes possible.[1] In principle (and before going into any detail), it stands to reason that one can harvest energy from the relative motion of two media (air and water) and use this energy in an engine. The question is whether this can be done "in practice." Figure 1 offers a "constructive proof of concept." Two propellers are mounted on the boat as shown. Assuming that the boat is moving forward, the water propeller—connected to an electricity generator—is dragged through the water with speed v_{boat}, generating electric power

$$P_{generated} = F_{drag} v_{boat}; \qquad (1)$$

we assume an ideal propeller and no losses. Here, F_{drag} refers to the drag on the propeller only — the drag on the hull is neglected.

[1] It is referred to as "dead downwind faster than the wind" (DDWFTTW), see http://www.skepticblog.org/2010/05/27/sailing-directly-downwind…-faster-than-the-wind/.

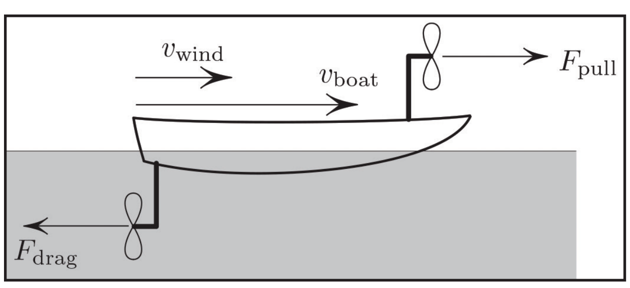

Figure 1. *Reference frame of the water. In steady motion, $F_{pull} = F_{drag} = F$. Here, F_{drag} is the drag on the propeller; the drag on the hull is ignored, as are other "imperfections."*

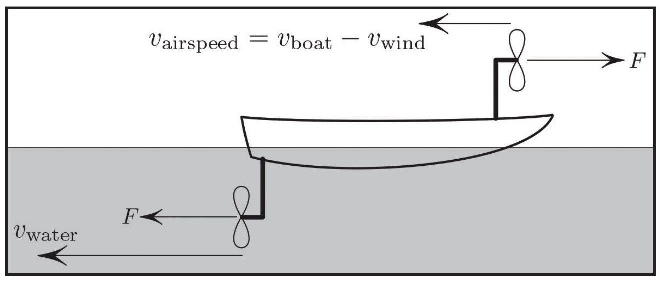

Figure 2. *Reference frame of the boat. Since $v_{water} > v_{airspeed}$, we have $P_{generated} > P_{consumed}$.*

The air propeller, on the other hand, pulls the boat forward and is driven by an electric motor, requiring power

$$P_{consumed} = F_{pull} v_{airspeed} = F_{pull}(v_{boat} - v_{wind}). \qquad (2)$$

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Does the power generated by the "dragger" suffice to feed the puller so as to maintain *constant speed* $v_{boat} > v_{wind}$? The answer is yes, because $F_{drag} = F_{pull}$ for constant speed, so that (1) and (2) imply

$$P_{generated} > P_{consumed}. \qquad (3)$$

Figure 2 gives an alternative view, from the boat's frame of reference; the key is that the oncoming water is faster than the oncoming air. And since the power generated/consumed depends on the propeller's speed relative to the medium, higher speed means greater power. The water propeller therefore generates more than the air propeller consumes.

Of course, the above idea is not limited to boats, and has been realized.[2]

A Solution to Last Month's Puzzle[3]

Refer to Figure 3's caption, which restates the puzzle. Imagine a firework exploding at point O in Figure 3, sending a myriad of shards in all directions, each with the same initial speed v. Ignoring the air resistance, the shards form an expanding circle[4] of radius vt at time t. And the center of this circle undergoes free fall, descending by $gt^2/2$ in time t. *The safety parabola is the envelope of this family of circles*, as illustrated in Figure 4. In other words, the safety ceiling serves a double duty as one envelope of two different families of curves. Because of this double role, any point T on the envelope is the point of tangency with a parabolic trajectory, and also with a circle (see Figure 5). Since the circle's center undergoes free fall, $OC = gt^2/2$, where t is the time of free fall. But this is the same t as the parabolic flight time from O to T, because only one shard ever reaches T. This completes the solution.

[2] https://www.wired.com/2010/06/downwind-faster-than-the-wind/

[3] *SIAM News, 50*(7), September 2017. https://sinews.siam.org/Details-Page/parabola-of-safety-and-the-jacobian

[4] We are considering a two-dimensional cross-section of the three-dimensional picture.

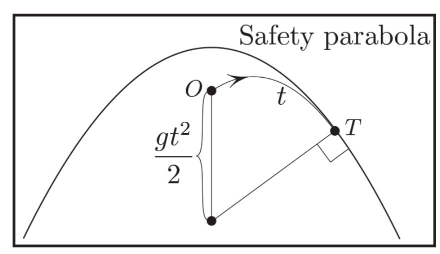

Figure 3. *Last month's puzzle. Show, without calculation, that the normal at the point of tangency T intersects the vertical at the distance $gt^2/2$ from the launch point O, where t is the time of flight from O to T.*

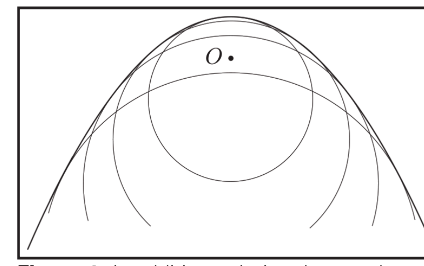

Figure 4. *In addition to being the envelope of a family of trajectories, the safety parabola is also the envelope of a one-parameter family of expanding circles with a free-falling center.*

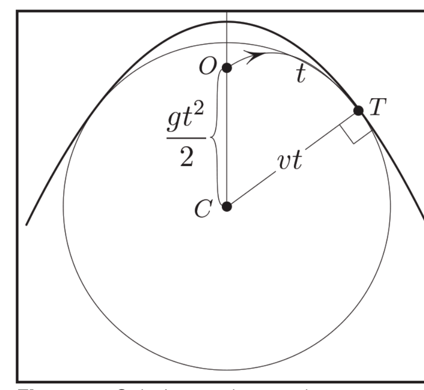

Figure 5. *Solution to the puzzle.*

The figures in this article were provided by the author.

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