The Moon Sine

MATHEMATICAL CURIOSITIES

By Mark Levi

The Sun-Moon-Eye Angle

On a long car ride from State College to Boston in late August, my wife and I were accompanied by a waning gibbous moon — a disk low on the horizon with a bit nibbled off, see Figure 1a. During the forced idleness of the long ride I realized how easy it is to tell the sun-moon-eye angle θ of Figure 2 from the face of the moon: namely,

$$\cos\theta = \frac{v}{r}, \qquad (1)$$

where v and r are marked in Figure 1; here, v may be either positive or negative, as stated in the caption. This sign convention gives an acute θ for the gibbous moon and an obtuse θ for the crescent moon, in agreement with common sense. Figure 3 explains the proof of (1). For the harvest moon, θ vanishes, and therefore so does sine on harvest moon.

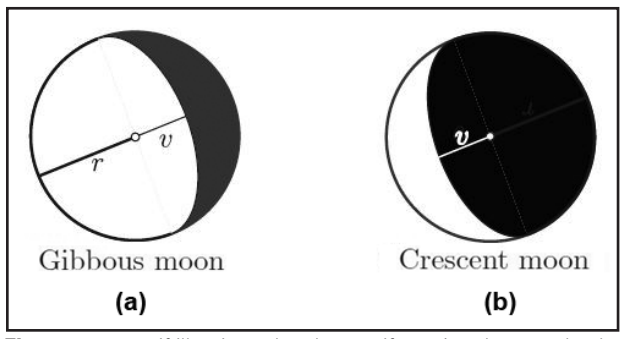

Figure 1. *$v>0$ if illuminated and $v<0$ if not. In other words, the positive direction is away from the sun.*

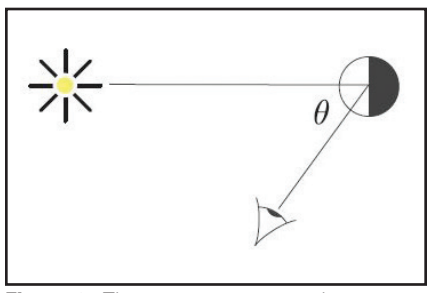

Figure 2. *The sun-moon-eye angle.*

The Terminator

The great circle on the moon that separates light from dark is called the lunar terminator. To our eye the terminator is an ellipse, since it is a parallel projection of a circle. Where are the foci of this ellipse? The answer is given by the same sun-moon-eye angle θ, as Figure 4 shows. And how do these foci move in time? It turns out that they execute harmonic motion if we neglect the eccentricities of the orbits of Earth and the moon. I leave out the proof of these claims.

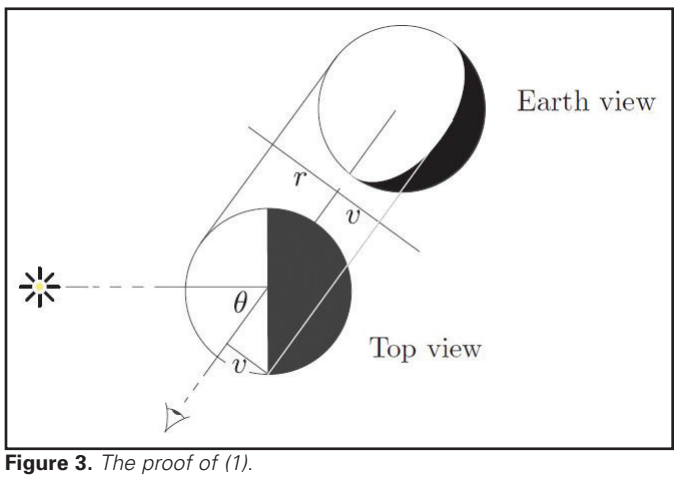

Figure 3. *The proof of (1).*

The Lunar Tilt Illusion

To conclude, I would like to mention a somewhat related Moon Tilt Illusion pointed out to me by Nick Trefethen: the tilt of the crescent seems wrong, and the moon should look fuller. Very nice discussions of this are available in [1] and [2].

The figures in this article were provided by the author.

References

[1] Berry, M.V. (2015). The squint Moon and the witch ball. *New J. Phys., 17*, 060201.

[2] Trefethen, L.N. (2011). The other moon illusion. In *Trefethen's Index Cards* (p. 270). Hackensack, NJ: World Scientific.

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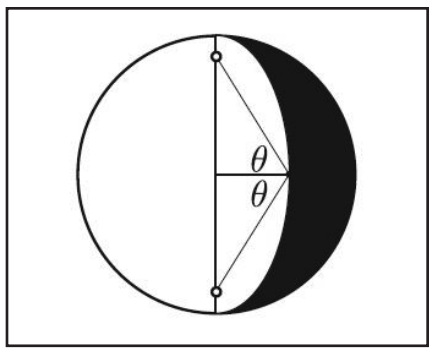

Figure 4. *The foci of the lunar terminator are given by the same θ as in Figure 2.*