A Water-based Solution of Polynomial Equations

MATHEMATICAL CURIOSITIES

By Mark Levi

I would like to describe a “hydrostatic” calculator of roots—at least of positive real ones—of a polynomial of any degree. As a pure thought experiment, let us cut the shapes shown in Figure 1 out of a sheet of weightless styrofoam. The $n\mathrm{th}$ shape displaces the volume x^n when submerged to depth x, provided the thickness of the sheet $=1$. Now our “calculator” (see Figure 2) consists of a weightless rod with the “origin” 0 marked on it. The styrofoam monomials can be affixed at any position on the rod. In addition, the trivial monomial $x^0=1$ is represented by a unit weight that can be slid to any position on the rod.

As an illustration, let us solve

$$ax^3-bx^2+cx-d=0, \qquad (1)$$

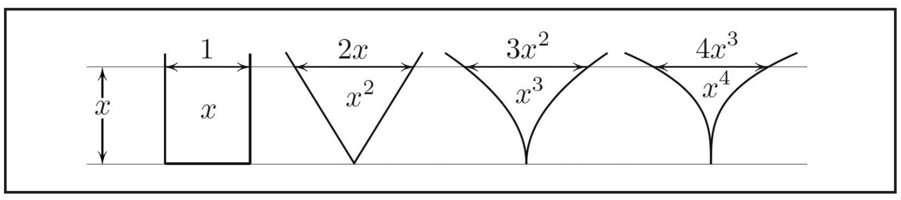

Figure 1. *Monomials incarnated.*

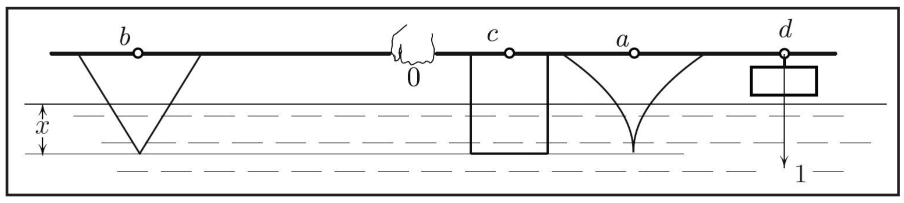

Figure 2. *Solving (1) by dunking the “scale.”*

with positive a, b, c, and d. We let these coefficients and the signs determine the locations of the monomials on the rod, as shown in Figure 2. Since it is not buoyant but weighty, the constant term follows the opposite rule: the minus sign in front of *d* places it to the right of 0.

With the “calculator” thus prepared, we hold the rod horizontally and slowly dunk it *until the torque we exert with the hand to keep the rod horizontal becomes zero*, i.e., until our scale balances. The depth x that provides this balance is a root of (1).

To understand why this method works, note that the polynomial ax^3-bx^2+cx-d is the torque relative to 0 of the forces acting upon the rod (see Figure 3). Therefore, the vanishing of the torque for a particular depth x amounts to x being a root of (1).

Of course, all of the above applies to polynomials of any degree, although this method only produces positive real roots.

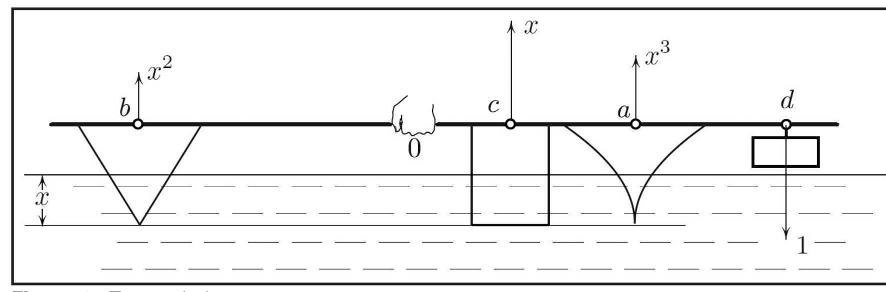

Figure 3. *Torque balance.*

The figures in this article were provided by the author.

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