Which Energy is Greater?

MATHEMATICAL CURIOSITIES

By Mark Levi

In the opening scene of the movie *Men in Black*, a large dragonfly splatters across the windshield of an oncoming truck. As a result of the collision, the insect increased its near-zero pre-collision velocity to that of the truck's and thus acquired kinetic energy. The collision also generated heat, as any inelastic collision does. Which of these two energies—kinetic or heat—is greater? The same question arises when a snowflake or rain droplet hits the windshield.

Interestingly, the two energies are equal (assuming that the dragonfly's pre-impact speed is negligible). To see why (and avoid calculation), let us examine the same event in two different frames of reference: first in the truck's and then in the ground observer's. In the truck's frame, the bug comes to a dead stop and slows from v (the truck's speed) to zero. All of its kinetic energy converts to heat,[1] so that acquired heat $H = mv^2/2$.

[1] Neglecting the small amount of energy that went into the sound, the windshield vibration, etc.

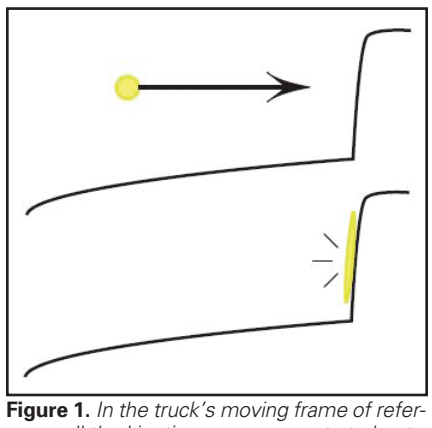

Figure 1. *In the truck's moving frame of reference, all the kinetic energy converts to heat.*

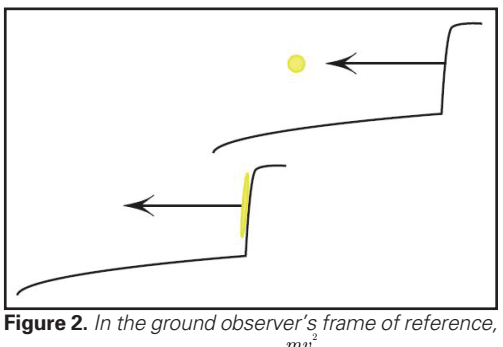

Figure 2. *In the ground observer's frame of reference, the acquired kinetic energy is* $\frac{mv^2}{2}$.

In the ground observer's frame, the dragonfly speeds up from zero to v, thus acquiring kinetic energy $K = mv^2/2$ so that $K = H$ as claimed. The energy split is fair; half goes into the disordered heat motion and half into the ordered translational motion.

A Puzzle

Imagine that the truck instead hits a rubber ball of the same mass as the insect that (like the insect) has zero pre-collision speed. Assuming a perfectly elastic collision, the ball speeds up from zero to $2v$. Indeed, in the truck's frame the ball goes from $-v$ to v, which corresponds to acceleration from zero to $2v$ in the ground frame. The ball therefore acquires kinetic energy $\frac{m(2v)^2}{2} = 4\frac{mv^2}{2}$. But this is more—exactly twice, in fact—than the total energy attained by the dragonfly! How can this be? Where did this extra $2\frac{mv^2}{2}$ come from?

The figures in this article were provided by the author.

Mark Levi (levi@math.psu.edu) is a professor of mathematics at the Pennsylvania State University.